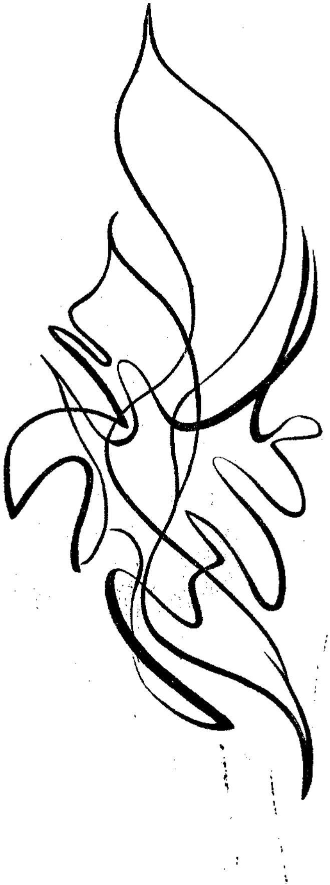

# WHAT IS APPLIED MATHEMATICS?

A. F. PILLOW

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# WHAT IS APPLIED MATHEMATICS?

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## INTRODUCTION

May I first say that I am happy to have found such a congenial Department of Mathematics in which to work on returning to Australia. I realize that much of the strength it has acquired over the last eight or so years is due, in no small measure and often in the face of adverse conditions, to the untiring leadership of its Head, Professor C. S. Davis.

An inaugural address for a mathematician is a particularly frustrating affair. For, with a non-technical audience, as I take this to be, he is robbed of the basic equations which he uses to express his thoughts about his subject and which are the prime ingredients of his medium of expression.

I have decided therefore not to attempt a serious description of the research problems in fluid dynamics and heat convection which interest me, but rather to discuss the role of mathematics in science and engineering and to investigate whether there is indeed a need for an independent discipline of applied mathematics. Patently, since I have raised the question, my thesis is that there is.

There is, I believe, a gulf developing between mathematics and the sciences which applied mathematics must strive to fill. I shall make an attempt to describe the characteristics of the ideal applied mathematician — the type of person who must fill this gap. In turn, this will lead me to discuss the type of training appropriate to an applied mathematician and its relationship to that of the pure mathematician, the scientist, and the engineer.

It will appear that to reach the creative level in applied mathematics, training at the graduate research level is vital and must be expanded. If time permits I should like to go on and, as an illustration, describe briefly some of the problems in fluid dynamics and heat convection which are being tackled by a research group which has been formed in the Department of Mathematics.

## The creators and users of mathematics

It seems worth while to me to attempt a classification into four different groups of those theoreticians who use mathematics to a significant extent. These groups are:

*The pure mathematicians:* Those concerned with the construction and deductive analysis of abstract, axiomatic systems and with the doing of algebra, analysis, geometry, and the like.

*The theoretical scientists:* Those primarily concerned with the discovery of new scientific laws, i.e. with the construction and understanding of idealized models which fit the experimental facts either in the exact sciences, the biological sciences, or the social sciences.

*The theoretical engineers:* Those concerned with the invention, design, understanding, and theory of maintenance of the constructs of applied science.

*The applied mathematicians:* Those concerned with the deductive analysis of the accepted idealized models of science and engineering, and with the use of mathematical methods to explore and describe phenomena in terms of these basic models. In addition, they are concerned with the light that these concrete realizations throw upon the development of new mathematical methods and abstract mathematical structures. In this classification, the statisticians would have to be housed primarily with the applied mathematicians.

The above divisions are not watertight and probably the designations would not be acceptable to all. Clearly there should be free interplay between all four at the mature level. Before we examine the relationship between these various theoreticians it would be pertinent to survey the role of mathematics in science and engineering.

## The role of mathematics in science and engineering

### *Scientific method*

Starting from the observed facts of nature, the scientist tries to fit them to a simple pattern. Properties and relations which roughly hold are noted and a classified system is built up into which further facts are fitted. Roughly speaking, the broader the class the more vaguely is it defined.

If the system becomes sufficiently complete and interesting it is usual to tidy up the reasoning and present it deductively in terms of conclusions drawn from an idealized model. The conclusions are then compared with experiment and, if the comparison is unsatisfactory, modification of the model is necessary. If, on the other hand, the comparison is satisfactory, the way is then open for generalization and extension of theory and experiment — and so the cycle of inductive and deductive reasoning repeats itself.

The question of the absolute reality of the model does not arise significantly; rather it is the simplicity of the model as viewed by our minds and its adequacy in accounting for the known and the as yet unknown experimental facts that are decisive. We may feel as Eddington did, that what determines the simplicity and adequacy of the model is in some measure a function of the structure of the human mind and of the manner of its stimulation by the external world. Figuratively speaking, Martians, even favourite ones, may well find a different set of models simpler and more convenient.

*Classical applied mathematics*

In those branches of science which can be adequately modelled, the part played by mathematics is mainly a deductive one. Broadly speaking, the whole subject is deduced by pure mathematics from the abstract properties and relationships of the entities of the model. Apart from the intrinsic interest, this approach has much to justify it. It presents the facts of the subject in a concise and assimilable form. Inconsistencies are revealed and further lines of possible development or generalization are made more apparent (e.g. Maxwell's theory of the non-stationary electromagnetic field).

Unfortunately, if the model is a detailed one, the general theory, when applied to a particular physical or engineering problem, usually leads to a highly complex mathematical question. Approximation then becomes necessary. In any case the model, being a simple abstraction from a large body of facts, will only fit the practical problem approximately. Approximations may be made either in the physical details of the model or in the mathematics. Usually it is a happy combination of the two that is required; the descriptive details of the model are modified in the light of previous mathematical and physical experience in such a way as to allow the desired analysis to be carried out and yet still maintain reasonable accord with the scientific facts.

*Mathematics as a scientific research tool*

The construction of suitable models is difficult and is far too easily neglected by both the mathematical and the scientific sides. Here the role of mathematics is a much more inductive one. We add features to the model experimentally keeping in mind that they must accord with the facts and make mathematical development possible. Comparison of the mathematical model with experiment leads either to modification of the model or else to more quantitative detail about its features. (Thus, in the kinetic theory of gases, we first assume a fairly general law of force between molecules and then by comparison of the mathematical consequences with experiment we deduce its quantitative nature.)

We try on the one hand to construct a model so that as many relevant scientific facts are encompassed as possible and yet on the other hand we endeavour to simplify the features sufficiently to make them yield to mathematical analysis. The process is one of intelligent trial and error, in which generalizations and speculations are made on both physical and mathematical grounds.

The complete analysis of any problem must not only account for the phenomena but must show just what features of the model are responsible for them and to what extent. Occasionally it is only after a complete mathematical analysis of a detailed model that the relevant features for the phenomena under investigation are revealed and the way is paved for a simpler, more direct theory. More often physical insight provides the model for the simpler theory and this pilot analysis breaks the path for the detailed mathematical analysis of the finer features.

The need for skill in the construction of models cannot be overemphasized. Often this construction is more difficult than the mathematical work that follows. Such skill is best gained by experience and from a knowledge of the relevant science, as well as from an understanding of the various simple standard models and an experience of their use in synthesis. Also, a grasp of the scope of the available mathematical methods is vitally necessary.

As a research tool, mathematics is at its best when the questions asked involve a range of cases — for example, the effect of varying certain parameters. For particular engineering problems an experimental approach is frequently more suited and accurate than a detailed and complex analysis of an inevitably idealized situation.

## Applied mathematics as an independent discipline

Now that we have surveyed the role of mathematics in science and engineering, the question that next arises is: Who is responsible for seeing that this role is carried out to the maximum effect? Do we need an applied mathematician, or will the pure mathematician, the scientist, and the engineer suffice? My main thesis tonight is, needless to say, that indeed we do need the applied mathematician.

In Australia and Britain I am tilting at windmills to some extent, for the subject is now well established. I count myself fortunate in having senior colleagues in Queensland who, through their wide experience and often too through their early training, understand well the role that this separate discipline plays.

However, in some of the major centres of the world, particularly in America, there is growing concern at the widening gulf that is developing between modern pure mathematics and the sciences. Modern developments in pure mathematics encourage the mathematicians to create new fields of activity which are exciting and aesthetically pleasing in their own right without reference to either general or specific scientific problems. The probability of such mathematics becoming applicable in reasonable time decreases. One cannot help feeling that mathematics, science, and engineering are basically a coherent body of intellectual pursuits which is subdivided because of human limitations. If intellectual hedonism is to be avoided, it becomes important to maintain an even development. The crucial step for this purpose is to emphasize the mutual interdependence of mathematics, the sciences, and their applications by recognizing applied mathematics as a legitimate self-propagating discipline of scientific activity, side by side with pure mathematics, the sciences, and engineering.

Such a statement immediately arouses doubts in the minds of many people. For the feeling is prevalent amongst certain groups that there are scientists and engineers who know how to use mathematics and that there are also pure mathematicians who apply their knowledge of mathematics to scientific and engineering problems. Are not these people applied mathematicians? Should applied mathematics exist as an independent discipline?

These are legitimate questions. If applied mathematicians are to consist *only* of the two categories just described, then applied mathematics cannot be regarded as a self-propagating discipline. Marriages will be consummated here and there but the progeny will be mules almost incapable of handing on the form thay have achieved. The subject would then lack an educational childhood in which its ideals and philosophies had been nurtured.

A challenging but practicable programme of education which introduces young students to such activities will always be needed. It is precisely the lack of an established undergraduate programme in applied mathematics in the United States which is causing the gulf so widely deplored by eminent scientists such as Professor C. C. Lin[1] of M.I.T. and Professor Mark Kac of the Rockefeller Institute.

## THE CHARACTERISTICS OF THE IDEAL APPLIED MATHEMATICIAN

Let us not talk of what applied mathematicians are but rather of what we would like to see in some ideal benign mirror held up to their minds and their activities.

Applied mathematics is a branch of science which seeks knowledge and understanding of the external physical universe in terms of idealized models and through use of mathematical methods and scientific inference. The ultimate goal of the efforts of the applied mathematician lies in the creation of ideas, concepts, and methods that are of basic and general applicability to the subject in question, be it fluid dynamics, meteorology, biochemistry, space engineering, information theory, or economics.

As a discipline of intellectual activity, applied mathematics lies between the sciences (e.g. physics, chemistry, or economics) and pure mathematics; in essence it represents an attitude, an approach, a way of thinking. The principal theme is the interdependence of mathematics and the sciences. A particularly challenging type of activity is the development of mathematical theories and adequate models in those scientific subjects which have not hitherto been subjected to

[1] Many of the ideas discussed, both in this and the next section of my lecture, are contained in an unpublished article by C. C. Lin (M.I.T. preprint 1963) "Education of Applied Mathematicians: A Program". I am grateful to him for some very helpful discussions on these topics. Similar ideas will be found in H. P. Greenspan, 1961, Applied Mathematics as a Science, *American Mathematical Monthly*, **68** (9) pp. 872–80.

systematic mathematical treatment. In turn these efforts lead by abstraction and generalization to new mathematical theories and ideas which are interesting in their own right as a part of pure mathematics. It is the recognition of this duality and a willingness to use the cross-fertilization of ideas in either direction that characterizes applied mathematics. The ideal applied mathematician must be a versatile scientist, a specialist in mathematics with a clear perspective and general knowledge of the fundamentals of a wide area of the sciences.

There are three important phases in the approach of an applied mathematician to a particular field of problems: First, he must formulate an idealized model in mathematical terms and then seed the area with a set of precisely formulated mathematical problems; secondly, he must solve the mathematical problems; and thirdly, he must discuss, interpret, and evaluate the results of his analysis. The solution of specific problems often serves merely as a focus and an aid in reaching final understanding. Successful solutions open up further paths of synthesis towards reality.

Pure mathematics arising from such problems, if shared with willing pure mathematicians, provides us with a means of bridging the gulf between mathematics and the sciences. Such mathematics has more chance of becoming applicable in other branches of science. Many pure mathematicians prefer to be free from the fetters of natural science and to become involved in a class of problems which are self-contained. This often enhances the aesthetic aspects of pure mathematics but can lead to a dangerous vacuum.

The basic difference in motivation between pure and applied mathematicians is reflected in the habits and practices of their activities. The applied mathematician must have a deep-seated love for the precision and economy of a rigorous mathematical demonstration but he cannot be made inactive by these loyalties. In the second phase of his activities his primary emphasis is always directed towards the ultimate solution and he frequently uses scientific reasoning to achieve this end. It is in this way that a feeling for the right emphasis is gained by plausible argument and an insight is acquired for approximations which will prove adequate in the mathematics. The applied mathematician's work is responsible and disciplined but he is not a deductive logician interested solely in the beauty of form and the power of abstraction. He must have enough background in pure mathematics to be able to distinguish between rigorous proof, reasonable demonstration, plausible argument, and hopeful speculation. There are times when he must be prepared to scrub up and determine with proper rigour the precise conditions under which results hold and indeed this is his ultimate goal.

In the first and third phases of his work the applied mathematician must follow the practices of the theoretical scientist and co-operate with him closely. The construction of an idealized mathematical model is certainly the most important and most difficult phase, and calls for a detailed knowledge of the observational and experimental facts related to the particular phenomenon under consideration. Penetrating insight and mature judgment founded on wide experience are required. In the third phase the applied mathematician must examine his results to reach a deeper understanding of the whole field. He must attempt to abstract the essentials and form concepts which are of wider applicability. It is at this stage that the conclusions must be checked against the existing body of knowledge and the predictions verified by further experiment and observation. If a model proves too complex to handle mathematically, he must be prepared to undress the model to its bare essentials whilst remembering the features removed. This he can often do in applied mathematics without getting his face slapped. With his coarsest glasses on, the applied mathematician can see only the three magnitudes, zero, one, and infinity. He will usually be prepared to consider various limiting cases in which the parameters become either zero or infinite or take on special values, even though these cases themselves may be unrealistic. His hope is, of course, that these limiting cases will lead to an eventual synthesis. Indeed, he is even prepared to dissect the model itself and, for the sake of an eventual synthesis, study sub-models in which certain physical effects, known to be important, are temporarily neglected.

Despite the similarities in activity there are subtle differences in attitude between a theoretical scientist and an applied mathematician. The theoretical physicist, for example, has his primary interest in the discovery of new physical laws while the applied mathematician, by comparison, places more emphasis on the use of mathematical methods for the description of physical phenomena in terms of known physical laws, and on the new mathematical ideas these problems stimulate. One might say that the difference lies in the relative extent to which inductive and deductive reasoning are emphasized in each discipline. Perhaps one can say that the applied mathematician builds up to the actual problem synthetically from mathematical sub-models while the theoretical scientist analyses the actual scientific problems into their basic phenomena.

The good applied mathematician must adapt his interests to the present and future vitality of the subject if his research efforts are to have an impact beyond the development of applicable mathematical methods. The desire and ability to cut across traditional scientific disciplines through the medium of mathematics are perhaps the main characteristics of an applied mathematician. It is inevitable, therefore, from this description, that the applied mathematician, who is wide-ranging, is forced to stay at the general fundamental level of a broad spectrum of science and to work with humility in co-operation with the scientific or engineering specialist.

## What type of training does an applied mathematician need?

### *Types of mathematical courses*

If one considers the needs of the various theoreticians that have been described, I believe that, for didactic purposes at least, three different types of mathematics courses are needed:

(i) pure courses,
(ii) applied courses,
(iii) service courses.

In fact, if we separate statistics and remember the special needs of school teachers, at least five different types of courses are necessary. Among the theoretical scientists and engineers are many who need a strong knowledge of conventional processes in mathematics, at the very least up to a reading level; for these people then there must be a service course available, taught by mathematicians with some practical experience of the mathematics needed and with an ability to exemplify it in the relevant field.

### *The training of an applied mathematician*

For the middle man, the applied mathematician, something more is needed, for he is responsible for pushing the analysis of physical models as far as mathematics will allow. He must then be solidly trained in the creative use of mathematical methods. This is his biggest threshold and I think he should therefore be housed with mathematicians. Since he will not primarily be analysing new mathematical discipline. Perhaps one can say that the applied mathematician builds up to the actual problem synthetically from mathematical sub-models while the theoretical scientist analyses the actual scientific problems into their basic phenomena.

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The second vital feature of an applied mathematician's training is, I feel, that he must *from the very beginning* be given adequate training in the deductive treatment of the classical models of science. A disciplined selection of the key models should be made with a representative spread and these courses, particularly at the honours level, should be taught by applied mathematicians who have gained some sympathy with and experience of the model-making side of applied mathematics.

If the syllabus is properly graded, it is certainly possible to teach applied mathematics along with pure and the relevant sciences in the first year at the university. Many universities in Britain and Australia do just this. The interplay of mathematical method with scientific intuition is healthy and by no means a one-way process. Point sources and multipoles were understood long before distributions and their derivatives were thought of. To postpone the treatment of applied mathematics to the last years of a mathematics course, as is done in some parts of North America, is as absurd as insisting that geometry cannot be studied until the algebra of transformations and their invariants is mastered.

As well as the pure and applied courses it is important that the student attend some of the relevant science or engineering courses and grasp the scientific facts or gain experience of practical engineering situations. This can certainly be achieved without overloading the timetable if the scientists or engineers are in sympathy and the student is allowed to forego some of the practical work, particularly in the second and later years.

I believe there is a pressing need for the honours training of some specialist engineers with an applied mathematician's outlook. With equal force it can be argued that a combined honours course in chemistry and applied mathematics would be mutually beneficial. If the students' loyalties are to be genuinely divided this interdisciplinary training must start at the undergraduate level.

One of the unsatisfactory features of an undergraduate course is that inevitably the synthetic model-making side of applied mathematics has to be postponed until the graduate years. For it is only then, in the physical sciences, that a sufficient arsenal of elementary standard models has been built up and analyzed mathematically. Perhaps this could be overcome in part by introducing essay-type questions during the early years in which the student could be asked, for example, to formulate and discuss a mathematical model for the game of billiards.

In an undergraduate course aimed at the physical sciences, an applied course must aim to cover, at a solid mathematical level, at least dynamics and the standard scalar, vector, and tensor fields of classical physics, i.e. heat conduction, continuum mechanics, and electrodynamics. In addition, there should be room at an optional level in the final year for a mathematical introduction to basic quantum mechanics, relativity, and statistical mechanics.

It is important nowadays that our students gain some detailed experience with the basic elementary non-deterministic models. Pure and applied courses in stochastic processes are needed, in which models, idealized from reality, are made for the changes of a statistical distribution with time. It is clear that already much of the applied mathematics of the future will relate to the social and biological sciences. Some of our students must be equipped to face this modelling challenge.

## Graduate work

It is at the graduate level that the vital and important teaching of applied mathematics at the creative level begins. It is at this level that there are very serious deficiencies in the Australian scene. Only 0.25 Doctoral theses per million population were produced in the whole of the mathematics and statistics departments throughout Australia during the period 1958-59, whereas for an equivalent period in the United States the figure was 1.4 per million.[2]

By comparison, in 1962-63, Toronto, a university of about 18,000 students, had 49 research students registered for higher degrees in mathematics, of whom 22 were candidates for degree of Doctor of Philosophy. These numbers do not arise from deficiencies in their undergraduate programme, for their honours course is a four year one and goes slightly further than our own four year course here.

The lack of production of creative workers in the field of mathematics is all the more serious when one considers the world-wide shortage and the large demand for mathematical doctorates in Australia. The academic demand has been established in Dr. Gani's statistical survey as an urgent and large one. When one considers the needs of the Council for Scientific and Industrial Research, the Weapons Research Establishment, and the Aeronautical Research Laboratories, and other State and Federal Government laboratories, as well as the increasing demand from the engineering and research sides of industry and the needs of the growing computer industry, it is obvious that the strong development of graduate work is a pressing necessity. Further, it appears that there is a sufficient flow of able students available to allow the development of this graduate work immediately. At present it is lack of attractive research opportunities that is siphoning off this supply into more routine duties.

The reasons for this lack of graduate development are not hard to find. Few of our top university professors can find any significant amount of time for professional work. They are continually called upon to deal with the governmental, administrative, and developmental tasks that are of vital importance to a growing university. We will always need the mature judgment and academic outlook of such self-sacrificing leaders, but it is vital that another echelon of full-time professional workers of comparable stature be established who can devote their energies full-time, without fear of loss of stature, to the training of research students and to the development of their subject, measured on the world scale. Fortunately with the coming of multiple professorships in departments and the appointment of permanent deans, such as at Monash University, these long overdue developments now seem possible.

[2] J. Gani, Trends in Mathematics at Australian Universities. *Vestes* 4 (1), March 1961, pp. 12-23 at p. 16.

At least collectively, they should expect, within the financial and other constraints of a problem, to be able to sway a wise dean and to have the right to appeal on contentious matters to the Vice-Chancellor. If the focus of responsibility for university academic development is shifted to the deans, then the overworked heads of departments will have more time to pursue the detailed and intensive development of their own departments.

Apart from coffee and tea and a good research library mathematics needs no costly apparatus. It flourishes primarily when a sufficient number, say three or four, of trained mathematicians with similar research interests are brought together and freed for a sizeable period of time from a major portion of their full routine work load. Research students are drawn to such an environment and can be matured rapidly in it, whilst they themselves provide a highly significant cross-fire of detail and critical opinion.

With this El Dorado in mind, we are aiming in the Mathematics Department at the grouping of research interests. Perhaps five or six groups would be about the right number. Each group would need about two to three post-doctoral staff to supervise and lead the research. With our present staff, this would call for only a fairly modest expansion at the senior lectureship and readership levels. The additional strength would permit the rotation of undergraduate lecturing duties so that a small proportion of the staff would always be available for full-time graduate work.

For the stability of the groups and the maintenance of a strong output it is essential that there be a steady flow of young full-time research workers. I believe that this can be ensured if the Department advertises for and appoints a regular number of adequately financed research students every year.

### The fluid dynamics and heat convection seminar

Detailed interest in weekly research seminars can be sustained if a central field of interest is selected and individual members are given specific problems to investigate under supervision.

To illustrate my remarks, let me describe briefly the work of a research group we formed during 1965 to study viscous flow and heat convection. This work has centred around a weekly seminar in which research problems and papers related to the main theme have been discussed. As well there are six members of the group, which is mainly composed of younger staff, who have started work on individual problems in the field and an attempt has been made to review and discuss with them individually the progress made each week. Most have already achieved some success but progress is inevitably slow owing to the near-to-full-time lecturing responsibilities of the members.

The theme of the seminar has been that viscous flow may be viewed as the diffusion and convection of local rotation ("vorticity") into the fluid from the boundaries where it is generated by the action of skin friction which ensures the fulfilment of the no-slip viscous boundary condition. The non-linearity of the problem arises from the fact that the diffused vorticity, in its final unknown equilibrium distribution, itself contributes a self-convection component to the total convection field in which it diffuses. Detailed preliminary studies are first being made of analogous linear problems in which scalar and vector quantities generated at simple point singularities diffuse in prescribed convection fields. It is hoped that local linearizations constructed along these lines will lead to a synthesis of the global solution of the non-linear problem by means of matching techniques. Similar problems arise in the study of rotating fluids.

Universities have a continuing duty to prune and develop their undergraduate syllabuses so that they are attuned to the present and future development of the subject. Not only must they hand on the scholarship of the past to their students but they must also train them in using this knowledge creatively. It is in this area that we must expand greatly, face the challenge, and develop new teaching methods. The traditional formal lecture no longer seems appropriate; rather it is that teacher and research student must join together and explore unsolved problems. Seminars, such as the fluid dynamics one, are needed in which the first tentative steps in exploring a problem are displayed and discussed in front of a group of research students who are working in the same field. Such methods demand an extravagant amount of the time of senior staff, but they are necessary if universities are to avoid becoming solely repositories for the knowledge of the past. We must enlarge our teaching horizons and place the emphasis squarely on post graduate training, on research, and on the using of knowledge creatively. It is for these ends that we must consciously train our top students.

SUMMARY

The answer to the question "What is applied mathematics?" is, I feel, that it is an independent scientific discipline which first seeks to construct idealized mathematical models in science and engineering by successive syntheses, and then aims, by deductive analysis of these models, to harness nature and gain a richer understanding of it.